EXAM ADVANCED LOGIC

April 9th, 2013

Instructions:

- Put you name and student number on the first page.
- Put your name on subsequent pages as well.
- Do not use pencil (or an erasable pen) or a red pen to make your exam.
- Motivate all your answers.
- If you want to inspect your exam after it has been graded, you can do so by making an appointment with Barteld Kooi.
- Please fill in the anonymous course evaluation.

Good luck!

1. **Induction (10 pt)** Let $\Pi(A)$ be the set of propositional parameters occurring in A. For example if $A = (p \wedge \neg q)$, then $\Pi(A) = \{p, q\}$.

   Now consider the sublanguage $\mathscr{L}_D$ of the language of propositional logic.

   i Each propositional parameter p is a formula of $\mathscr{L}_D$.

   ii If A is a formula of $\mathscr{L}_D$, then so is $\neg A$.

   iii If A and B are formulas of $\mathscr{L}_D$ such that $\Pi(A) \cap \Pi(B) = \emptyset$, then so is $(A \wedge B)$.

   iv Nothing is a formula of $\mathscr{L}_D$ unless it is generated by repeated applications of i, ii and iii.

   Prove by induction that for each formula A of $\mathscr{L}_D$ both A and $\neg A$ are satisfiable. (A formula A is satisfiable iff there exists a valuation $v : P \rightarrow \{0, 1\}$ such that $v(A) = 1$.)

2. **Three-valued logics (10 pt)** Show that for all formulas A and B

$$A \models_{RM_3} B \text{ iff } \neg B \models_{K_3} \neg A$$

3. **FDE tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in **FDE**. If the inference is invalid, provide a counter-model.

$$\neg p \vee q, \neg(r \wedge s) \vdash (\neg p \vee s) \vee (\neg r \vee q)$$

   NB: Do not forget to draw a conclusion from the tableau.

4. **Fuzzy logic (10 pt)** Determine whether the following holds in $Ł_{\aleph}$ (where $D = \{1\}$). If so, show that if the premises have value 1, so does the conclusion. If not, provide a counter-model.

$$p \vee q \models q \rightarrow (p \rightarrow q)$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in K. If the inference is invalid, provide a counter-model.

$$\Box\neg p, \Diamond(p \vee \Diamond q) \vdash_K \Diamond\Diamond(q \wedge p)$$

   NB: Do not forget to draw a conclusion from the tableau.

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in $K^t_{\tau\phi\beta}$. If the inference is invalid, provide a counter-model.

$$p \vdash_{K_{\delta\sigma}} \Box\Diamond\Diamond p$$

   NB: Do not forget to draw a conclusion from the tableau.

7. **Soundness and completeness (10pt)** Consider the following branch of a tableau b:

$$\begin{array}{c} \Diamond p, 0 \\ \Box(q \wedge \Diamond p), 0 \\ \neg\Box(p \wedge q), 0 \\ 0r1 \\ p, 1 \\ \Diamond\neg(p \wedge q), 0 \\ 0r2 \\ \neg(p \wedge q), 2 \end{array}$$

Consider the following model $I = \langle W, R, v \rangle$:

$$\begin{array}{lcl} W & = & \{w_1, w_2\} \\ R & = & \{\langle w_1, w_1 \rangle, \langle w_1, w_2 \rangle, \langle w_2, w_1 \rangle\} \\ v_{w_1}(p) & = & 0 \\ v_{w_1}(q) & = & 0 \\ v_{w_2}(p) & = & 1 \\ v_{w_2}(q) & = & 1 \end{array}$$

Show that I is faithful to b. (This means you have to provide a function $f : \mathbb{N} \to W$ and show that it has the desired properties.)

8. **First-order modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in CK. If the inference is invalid, provide a counter-model.

$$\Box\neg\exists x(Ax \wedge Bx), \exists x\Diamond(Cx \wedge Ax) \vdash_{CK} \Diamond\exists x(Cx \wedge \neg Bx)$$

NB: Do not forget to draw a conclusion from the tableau.

9. **Default logic (10 pt)** Consider the folowing set of default rules:

$$D = \{d_1 = \frac{p : q}{q}, \qquad d_2 = \frac{p : q \wedge r}{s \wedge r}, \qquad d_3 = \frac{q : \neg r}{s \wedge \neg r}\},$$

and initial set of facts:

$$W = \{p\}.$$

Recall that a formula φ is a *skeptical consequence* of (W, D) if and only if φ is true in every extension of (W, D), while it is a *credulous consequence (goedgelovig gevolg)* of (W, D) if and only if φ is true in some extension of (W, D).

(a) Draw the process tree of this default theory.

(b) Is $\neg r$ a skeptical consequence of this theory?

(c) Is $q \wedge \neg r$ a credulous consequence of this theory?